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ANALYSIS OF DETERIORATING INVENTORY MODEL FOR TRAPEZOIDAL TYPE DEMAND RATE

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ABSTRACT: At the beginning of the season it increases, in the mid of the season it decreases and becomes asymptotic. In the literature of inventory after the development of classical economic order quantity model researchers extensively studied several aspects of inventory modeling by assuming constant demand rate. Trapezoidal type demand pattern is generally seen in the case of any fad or seasonal goods coming to market. The demand rate for such items increases with the time up to certain time and then ultimately stabilizes and becomes constant, and finally the demand rate approximately decreases to a constant, and finally the demand rate approximately decreases to a constant or zero, and the begins the next replenishment cycle.

KEYWORDS: Demand rate, economic order.

INTRODUCTION

Generally, the researchers assumed the time varying demand as an increasing or decreasing function of time, however in reality; this hypothesis is not appropriate for all products. One can observe the two-fold ramp type pattern in demand for items like fashion apparel, particular kind of eatables and festival accessories having restricted phase for sales and becomes out of use at end of phase. Such structure is known as ‘‘trapezoidal ramp-type’’. In opening phase, the demand rises up to a certain time point and becomes constant later but starts falling towards the end phase. Hill (1995) was the first to consider ramp type demand through inventory modelling. After that, various researchers have drawn considerable attention to exploration of ramp-type demand. Mandal and Pal (1998) proposed the inventory model with ramp-type demand for exponentially deteriorating items with shortages. Wu and Ouyang (2000) developed EOQ models for different replenishment policies; one of which included shortage at beginning while the second considered shortage at later phase. Continuing in same manner, Wu (2001) investigated a model for deteriorating items with ramp-type demand and partial backlogging. Giri et al. (2003) studied an extension for ramp-type demand inventory model with Weibull distribution deterioration rate. Manna and Chaudhuri (2006) developed a production model with ramp-type two time periods regarded as demand form taking demand dependent production into account. Deng et al. (2007) got a note published on the uncertain results found by Mandal and Pal (1998) and Wu and Ouyang (2000) and proved a more reliable solution. Panda et al. (2008, 2009) worked on Giri et al.’s (2003) model to change one-fold demand model to two-fold demand. Hill‘s (1995) model was extended to trapezoidal-type demand rate by Cheng and Wang (2009). Panda et al. (2009) developed the inventory model with quadratic ramp-type demand function which determined the optimal production stopping time. Skouri et al. (2009) extended the model of Deng et al. (2007) to wide-ranging ramp-type demand rate, Weibull distribution deterioration rate, and general partial backlogging rate. Further, this model was extended by Hung (2011). He incorporated arbitrary component in ramp-type demand pattern. Shah and Shah (2012) investigated a model for items having trapezoidal demand in an integrated system. Inflation plays a vital role in the inventory system and production management though the decision makers may face difficulties in arriving at answers related to decision making. At present, it is impossible to ignore the effects of inflation and it is necessary to consider the effects of inflation on the inventory system. The application of this concept was initiated by Buzacott (1975), who developed an EOQ model with inflation subject to different types of pricing policies. After that, several researchers like Brahmbhatt (1982), Chandra and Bahner (1985), Datta and Pal (1991), Hariga (1995, 1996) and Ray and Chaudhuri (1997) explored the effect of inflation and time value of money on inventory models. Liao et al. (2000) developed an inventory model for initial-stock dependent consumption rate with permissible delay in payment under the inflationary environment. Moon et al. (2005) developed the model for ameliorating/deteriorating items with time varying demand pattern over a finite planning horizon and explored the effects of inflation and time value of money. Jolai et al. (2006) studied the system of perishable items for stock dependent demand and shortages with inflation. Dey et al. (2008) developed a finite time horizon inventory problem for deteriorating items for a two warehouse system under inflation and time value of money. Roy et al. (2009) worked on inventory model for a deteriorating item with displayed stock dependent demand under fuzzy inflation and time discounting over a random planning horizon. Yang et al. (2010) discussed a model under similar environment relaxing the warehouse situation. Das et al. (2011) developed an economic production lot size model for an item with imperfect quality by considering random machine failure. Recently, Guria et al. (2013) worked on a deterministic and random planning horizon based model under inflation considering both the cases of shortage as well as no shortage.

REVIEW OF LITERATURE

Donaldson (1977) gave full analytic solution to the economic order quantity model for linearly increasing time dependent demand rate. Since then, several aspects of inventory control have been examined in different scenarios under the assumption of time varying demand pattern. Four types of time dependent demand patterns are found in the literature: (i) linear positive/negative (Goyal, 1986; Hariga, 1995) (ii) exponentially increasing/decreasing (Hariga & Bankherouf, 1994) (iii) quadratic or parabolic and (iv) ramp-type. Linear trend in demand indicates uniform change in demand rate per unit time. Quadratic time dependent demand rate represents accelerated growth or decline in the demand rate. Hill (1995) first proposed a time dependent demand pattern by considering it as the combination of two different types of demand such as increasing demand followed by a constant demand in two successive time periods over the entire time horizon and termed it as ramp-type time dependent demand pattern. He derived the exact solution to compare with the Silver-Meal heuristic. Mandal and Pal (1998) extended the inventory model with ramp type demand for deteriorating items and allowing shortage. Wu and Ouyang (2000) extended the inventory model to include two different replenishment policies: (a) models starting with no shortage and (b) models starting with shortage. Wu (2001) further investigated the inventory model with ramp type demand rate such that the deterioration followed the Weibull distribution deterioration and partial backlogging. Giri *et al.* (2003) extended the ramp type demand inventory model with a more generalized Weibull deterioration distribution. Khanra and Chaudhuri (2003) proposed quadratic type of demand pattern to diminish extraordinarily high rate of change in demand found to occur for exponential time dependent demand. All time dependent demand patterns reported above are unidirectional, i.e., increases continuously or decreases continuously or constant. On the other hand, researches on the real market oriented time dependent demand is very restrictive. Manna and Chaudhuri (2006) have developed a production inventory model with ramp-type two time periods classified demand pattern where the finite production rate depends on the demand. The demand increases linearly with time in the first period of time, and then it becomes steady for the remaining time of the production cycle. They noted that this type of demand pattern is generally followed by new brand of consumer goods coming to the market. Deng *et al.* (2007) point out some questionable results of Mandal and Pal (1998) and Wu and Ouyang (2000), and then resolved the similar problem by offering a rigorous ad efficient method to derive the optimal solution. In (2008) Cheng and Wang extended the Hill's ramp type demand rate to trapezoidal type demand rate.

In this paper, we study an inventory model for determining the optimal replenishment schedule for decaying items under partially backlogged shortages, in which the inventory deteriorates at a variable rate over time and trapezoidal type demand rate, that is the demand rate is a piecewise linearly function are considered. Backlogging rate is an exponential decreasing function of time. The model is developed over a finite time horizon to determine the optimal replenishment policy by minimizing total system cost of the system. Finally, numerical examples on each model highlighting the results obtained are given and sensitivity analysis on some parameters is made. The variations in the system statistics with a variation in system parameters has also been illustrated graphically.

ASSUMPTIONS:

1.The replenishment rate is infinite, thus replenishment rate is instantaneous.

2.The demand rate, D(t) which is positive and consecutive, is assumed to be a trapezoidal type function of time that is:

$$D(t) = \begin{cases} a_1 + b_1 t, t \le \mu_1 \\ D_0, \mu_1 \le t \le \mu_2 \\ a_2 - b_2 t, \mu_2 \le t \le T \le \dfrac{a_2}{b_2} \end{cases}$$

Where μ_1 is time point changing from the increasing linearly demand to constant demand, and μ_2 is time point changing from the constant demand to the decreasing linearly demand.

3. The length of each ordering cycle is fixed.

4. Deterioration rate is taken as time dependant.

5. Shortages occur and partially backlogged.

6. Backlogging rate is exponential decreasing function of time.

NOTATIONS:

I(t)	inventory level at any time t
T	the fixed length of each ordering cycle
Kt	Time dependent deterioration rate and K is a constant
t_1	the time when the inventory level reaches zero
A_0	fixed ordering cost per order
c_1	the cost of each deteriorated item
c_2	inventory holding cost per unit per unit of time
c_3	shortage cost per unit per unit of time
S	maximum inventory level
Q	ordering quantity per cycle
μ_1	time point changing from the increasing linearly demand to constant demand
μ_2	time point changing from the constant demand to the decreasing linearly demand
$e^{-\delta t}$	waiting time during shortages up-to next replenishment

MATHEMATICAL FORMULATION:

We considered an order level inventory model with trapezoidal type demand rate. Replenishment occurs at time t = 0 when the inventory level attains its maximum. From t = 0 to t_1, the inventory level reduces due to demand and deterioration. At t_1, the inventory level achieves zero, then shortage is allowed to occur during the time interval (t_1, T) and all of the demand during the shortage period is partially backlogged due to impatience of customer.

The differential equations governing the transition of the system are given by:

$$\frac{dI(t)}{dt} = -KtI(t) - D(t) \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -e^{-\delta t} D(t) \quad t_1 \le t \le T \tag{2}$$

With boundary condition $I(t_1)=0$

Now we consider three possible cases based on the values of t_1, μ_1 and μ_2. These three cases are shown as follows:

Case-1: When $0 \le t_1 \le \mu_1$

Due to combined effect of demand and deterioration the inventory level gradually diminishes during the period $[0, t_1]$ and ultimately falls to zero at time t_1. The differential equations are given by:

$$\frac{dI(t)}{dt} = -KtI(t) - (a_1 + b_1 t) \quad 0 \le t \le t_1 \tag{3}$$

$$\frac{dI(t)}{dt} = -(a_1 + b_1 t)e^{-\delta t} \quad t_1 \le t \le \mu_1 \tag{4}$$

$$\frac{dI(t)}{dt} = -D_0 e^{-\delta t} \quad \mu_1 \le t \le \mu_2 \qquad ..(5)$$

$$\frac{dI(t)}{dt} = -(a_2 - b_2 t)e^{-\delta t} \quad \mu_2 \le t \le T \qquad ..(6)$$

With boundary condition $I(t_1) = 0$

The solutions of these equations are given by:-

$$I(t) = [a_1(t_1 - t) + \frac{b_1}{2}(t_1^2 - t^2) + \frac{a_1 k}{6}(t_1^3 - t^3) + \frac{b_1 k}{8}(t_1^4 - t^4)]e^{-\frac{kt^2}{2}} \quad 0 \le t \le t_1 \qquad .(7)$$

$$I(t) = (a_1 + b_1 t)\frac{e^{-\delta t}}{\delta} - (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta} + \frac{b_1}{\delta^2}(e^{-\delta t} - e^{-\delta t_1}) \quad t_1 \le t \le \mu_1 \tag{8}$$

$$I(t) = \frac{D_0}{\delta}(e^{-\delta t} - e^{-\delta \mu_1}) + (a_1 + b_1\mu_1)\frac{e^{-\delta\mu_1}}{\delta} - (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta} + \frac{b_1}{\delta^2}(e^{-\delta\mu_1} - e^{-\delta t_1}) \quad \mu_1 \le t \le \mu_2 \tag{9}$$

$$I(t) = (a_2 - b_2 t)\frac{e^{-\delta t}}{\delta} - (a_2 - b_2 T)\frac{e^{-\delta t}}{\delta} + \frac{b_2}{\delta^2}(e^{-\delta T} - e^{-\delta t}) \quad \mu_2 \le t \le T \tag{10}$$

The inventory level at the beginning is given by:-

$$I(0) = [a_1 t_1 + \frac{b_1}{2}t_1^2 + \frac{a_1 k}{6}t_1^3 + \frac{b_1 k}{8}t_1^4] = S \tag{11}$$

The total no. of items which perish in the inventory $[0, t_1]$, say D_T, is:-

$$D_T = S - \int_0^{t_1} D(t).dt$$

So the cost of deterioration is given by:-

$$\text{Det. Cost} = (\frac{a_1 k}{6}t_1^3 + \frac{b_1 k}{8}t_1^4)c_1 \tag{12}$$

The total number of inventory carried during the interval [$0, t_1$]:

$$H_T = c_2 \int_0^{t_1} I(t)dt$$

$$= c_2\left(a_1 \frac{t_1^2}{2} + b_1 \frac{t_1^3}{3} + \frac{a_1 k}{12} t_1^4 + \frac{b_1 k}{15} t_1^5\right) \qquad (13)$$

The total shortage quantity during the interval $[t_1, T]$, say B_T, is:-

$$B_T = -\int_{t_1}^{\mu_1} I(t).dt - \int_{\mu_1}^{\mu_2} I(t).dt - \int_{\mu_2}^{T} I(t).dt$$

$$\begin{aligned} B_T = \; & -\{(a_1 + b_1\mu_1)\frac{e^{-\delta\mu_1}}{-\delta^2} - \frac{b_1 e^{-\delta\mu_1}}{\delta^3} - (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta}\mu_1 - \frac{b_1}{\delta^2}\left(\frac{e^{-\delta\mu_1}}{\delta} + \mu_1 e^{-\delta t_1}\right) + \\ & (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta^2} + \frac{b_1 e^{-\delta t_1}}{\delta^3} + (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta} t_1 + \frac{b_1}{\delta^2}\left(\frac{e^{-\delta t_1}}{\delta} + t_1 e^{-\delta t_1}\right)\} - \\ & \{\frac{D_0}{\delta}\left(\frac{e^{-\delta\mu_2}}{-\delta} - \mu_2 e^{-\delta\mu_1}\right) + (a_1 + b_1\mu_1)\frac{e^{-\delta\mu_1}}{\delta}\mu_2 - (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta}\mu_2 + \frac{b_1}{\delta^2}(e^{-\delta\mu_1} - e^{-\delta t_1})\mu_2 + \\ & \frac{D_0}{\delta}\left(\frac{e^{-\delta\mu_1}}{\delta} + \mu_1 e^{-\delta\mu_1}\right) - (a_1 + b_1\mu_1)\frac{e^{-\delta\mu_1}}{\delta}\mu_1 + (a_1 + b_1 t_1)\frac{e^{-\delta t_1}}{\delta}\mu_1 - \frac{b_1}{\delta^2}(e^{-\delta\mu_1} - e^{-\delta t_1})\mu_1\} \\ & -\{(a_2 - b_2 T)\frac{e^{-\delta T}}{-\delta^2} + \frac{b_2 e^{-\delta T}}{\delta^3} - (a_2 - b_2 T)\frac{e^{-\delta T}}{\delta}T + \frac{b_2}{\delta^2}\left(e^{-\delta T}T + \frac{e^{-\delta T}}{\delta}\right) + \\ & (a_2 - b_2\mu_2)\frac{e^{-\delta\mu_2}}{\delta^2} - \frac{b_2 e^{-\delta\mu_2}}{\delta^3} + (a_2 - b_2 T)\frac{e^{-\delta T}}{\delta}\mu_2 - \frac{b_2}{\delta^2}\left(e^{-\delta T}\mu_2 + \frac{e^{-\delta\mu_2}}{\delta}\right)\} \end{aligned} \qquad (14)$$

Shortage Cost = $C_3 B_T$ (15)

Then, the average total cost per unit time under the condition $t_1 \le \mu_1$ can be given by:

$$C_1(t_1, T) = \frac{1}{T}[A_0 + c_1 D_T + c_2 H_T + c_3 B_T] \qquad (16)$$

The necessary conditions for the total relevant cost per unit time of the equation (16) is to be minimized is

$$\frac{\partial C_1(t_1, T)}{dt_1} = 0 \text{ and } \frac{\partial C_1(t_1, T)}{dT} = 0$$

$$\left.\frac{\partial^2 C_1(t_1, T)}{\partial t_1^2}\right| > 0, \left.\frac{\partial^2 C_1(t_1, T)}{\partial T^2}\right| > 0$$

$$\left.\left(\frac{\partial^2 C_1(t_1, T)}{\partial t_1^2}\right)\left(\frac{\partial^2 C_1(t_1, T)}{\partial T^2}\right) - \left(\frac{\partial^2 C_1(t_1, T)}{\partial t_1 \partial T}\right)\right| > 0 \qquad (17)$$

Case 2: $\mu_1 \le t_1 \le \mu_2$

The differential equations governing the transition of the system are given by:-

$$\frac{dI(t)}{dt} = -KtI(t) - (a_1 + b_1 t) \quad 0 \le t \le \mu_1 \qquad (18)$$

$$\frac{dI(t)}{dt} = -KtI(t) - D_0 \quad \mu_1 \le t \le t_1 \qquad (19)$$

$$\frac{dI(t)}{dt} = -D_0 e^{-\delta t} \quad t_1 \le t \le \mu_2 \qquad (20)$$

$$\frac{dI(t)}{dt} = -(a_2 - b_2 t)e^{-\delta t} \quad \mu_2 \le t \le T \qquad (21)$$

The beginning inventory level is:-

$$I(0) = S = \{a_1\mu_1 + \frac{a_1 K}{6}\mu_1^3 + \frac{b_1\mu_1^2}{2} + \frac{b_1 K\mu_1^4}{8} + D_0(t_1 + \frac{K}{6}t_1^3)\} \qquad (22)$$

The total no. of items which perish in the inventory [$0, t_1$], say D_T, is:-

$$D_T = S - \int_0^{t_1} D(t).dt$$

So the cost of deterioration is given by:

$$D.C. = \{a_1\mu_1 + \frac{a_1K}{6}\mu_1^3 + \frac{b_1\mu_1^2}{2} + \frac{b_1K\mu_1^4}{8} + D_0(t_1 + \frac{K}{6}t_1^3) - (a_1\mu_1 + \frac{b_1\mu_1^2}{2}) - D_0(t_1 - \mu_1)\}c_1 \tag{23}$$

The total number of inventory carried during the interval $[0, t_1]$: -

$$H_T = \int_0^{t_1} I(t)dt$$

$$= [a_1\frac{\mu_1^2}{2} + a_1\frac{K\mu_1^4}{8} + \frac{b_1}{3}\mu_1^3 + b_1\frac{K\mu_1^5}{10} + D_0\{\mu_1(t_1-\mu_1)\} + \frac{K}{6}\{\mu_1(t_1^3 - \mu_1^3)\}$$
$$-\frac{a_1K}{24}\mu_1^4 - \frac{b_1K}{30}\mu_1^5 - D_0\frac{K}{2}\{\frac{\mu_1^3}{3}(t_1-\mu_1)\} + D_0(\frac{t_1^2}{2} + \frac{K}{8}t_1^4) - \frac{D_0K}{24}t_1^4 \tag{24}$$
$$-D_0\{(t_1\mu_1 - \frac{\mu_1^2}{2}) + \frac{K}{6}(t_1^3\mu_1 - \frac{\mu_1^4}{4})\} + \frac{D_0K}{2}(t_1\frac{\mu_1^3}{3} - \frac{\mu_1^4}{4})]$$

The total shortage quantity during the interval $[t_1, T]$, say B_T, is:-

$$B_T = -\int_{t_1}^{\mu_2} I(t).dt - \int_{\mu_2}^{T} I(t).dt$$

$$B_T =$$
$$[\frac{D_0}{\delta}(\frac{e^{-\delta\mu_2} - e^{-\delta t_1}}{\delta} + e^{-\delta t_1}(\mu_2 - t_1)) + (a_2 - b_2T)\frac{e^{-\delta T}}{\delta^2} - (a_2 - b_2\mu_2)\frac{e^{-\delta\mu_2}}{\delta^2}$$
$$-2\frac{b_2}{\delta^3}(e^{-\delta T} - e^{-\delta\mu_2}) + (a_2 - b_2T)(T-\mu_2)\frac{e^{-\delta T}}{\delta} - \frac{b_2}{\delta^2}e^{-\delta T}(T-\mu_2)] \tag{25}$$

Then the average total cost per unit time for this case is given by:-

$$C_2(t_1) = \frac{1}{T}[A_0 + c_1D_T + c_2H_T + c_3B_T] \tag{26}$$

The necessary conditions for the total relevant cost per unit time of the equation (30) is to be minimized is

$$\frac{\partial C_2(t_1,T)}{dt_1} = 0 \text{ and } \frac{\partial C_2(t_1,T)}{dT} = 0$$
$$\left.\frac{\partial^2 C_2(t_1,T)}{\partial t_1^2}\right| > 0, \left.\frac{\partial^2 C_2(t_1,T)}{\partial T^2}\right| > 0$$
$$\left.\left(\frac{\partial^2 C_2(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 C_2(t_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 C_2(t_1,T)}{\partial t_1 \partial T}\right)\right| > 0 \tag{27}$$

Case-3: $\mu_2 \le t_1 \le T$

The differential equations governing the transition of the system are given by:-

$$\frac{dI(t)}{dt} = -KtI(t) - (a_1 + b_1t) \quad 0 \le t \le \mu_1 \tag{28}$$

$$\frac{dI(t)}{dt} = -KtI(t) - D_0 \quad \mu_1 \le t \le \mu_2 \tag{29}$$

$$\frac{dI(t)}{dt} = -KtI(t) - (a_2 - b_2t) \quad \mu_2 \le t \le t_1 \tag{30}$$

$$\frac{dI(t)}{dt} = -(a_2 - b_2t)e^{-\delta t} \quad t_1 \le t \le T \tag{31}$$

Using boundary condition $I(t_1) = 0$

Solution of these equation are given by-

$$I(t)=\{S-(a_1t+\frac{b_1}{2}t^2+\frac{a_1K}{6}t^3+\frac{b_1K}{8}t^4)\}e^{-\frac{Kt^2}{2}} \quad 0\le t\le \mu_1 \tag{32}$$

$$I(t)=\{D_0((\mu_2-t)+\frac{K}{6}(\mu_2{}^3-t^3))+a_2(t_1-\mu_2)+\frac{a_2K}{6}(t_1{}^3-\mu_2{}^3)-\frac{b_2}{2}(t_1{}^2-\mu_2{}^2)-\frac{b_2K}{8}(t_1{}^4-\mu_2{}^4)\}e^{-\frac{Kt^2}{2}}$$
$$\mu_1\le t\le \mu_2 \tag{33}$$

$$I(t)=\{a_2(t_1-t)+\frac{a_2K}{6}(t_1{}^3-t^3)-\frac{b_2}{2}(t_1{}^2-t^2)-\frac{b_2K}{8}(t_1{}^4-t^4))\}e^{-\frac{Kt^2}{2}}\} \quad \mu_2\le t\le t_1 \tag{34}$$

$$I(t)=(a_2-b_2t)e^{-\frac{\delta t}{\delta}}-(a_2-b_2t_1)e^{-\frac{\delta t_1}{\delta}}-\frac{b_2}{\delta^2}(e^{-\delta t}-e^{-\delta t_1}) \quad t_1\le t\le T \tag{35}$$

$$\begin{aligned}S&=D_0\{(\mu_2-\mu_1)+\frac{K}{6}(\mu_2{}^3-\mu_1{}^3)\}+a_2(t_1-\mu_2)+\frac{a_2K}{6}(t_1{}^3-\mu_2{}^3)\\&-\frac{b_2}{2}(t_1{}^2-\mu_2{}^2)-\frac{b_2K}{8}(t_1{}^4-\mu_2{}^4)\}+(a_1\mu_1+\frac{a_1K}{6}\mu_1{}^3+b_1\frac{\mu_1{}^2}{2}+\frac{b_1K}{8}\mu_1{}^4)\end{aligned} \tag{36}$$

The total number of items which deteriorate in the interval [0, t_1] is

$$D_T=S-\int_0^{t_1}D(t)dt$$

$$D_T=S-\{(a_1\mu_1+b_1\frac{\mu_1{}^2}{2})-D_0(\mu_2-\mu_1)-(a_2t_1-b_2\frac{t_1{}^2}{2}-\mu_2a_2+b_2\frac{\mu_2{}^2}{2})\} \tag{37}$$

The total number of inventory carried during the interval [$0,t_1$] :

$$H_T=\int_0^{t_1}I(t)dt$$

$$H_T=\begin{aligned}&\{S\mu_1-(a_1\frac{\mu_1{}^2}{2}+\frac{a_1K}{24}\mu_1{}^4+\frac{b_1}{6}\mu_1{}^3+\frac{b_1K}{40}\mu_1{}^5)-\frac{KS}{6}\mu_1{}^3+\frac{K}{2}(a_1\frac{\mu_1{}^4}{4}+\frac{b_1}{10}\mu_1{}^5)+\{D_0(\frac{\mu_2{}^2}{2}+\frac{K}{8}\mu_2{}^4)+\\&a_2(t_1-\mu_2)(\mu_2-\mu_1)+\frac{a_2K}{6}(t_1{}^3-\mu_2{}^3)(\mu_2-\mu_1)-\frac{b_2}{2}(t_1{}^2-\mu_2{}^2)(\mu_2-\mu_1)-\frac{b_2K}{8}(t_1{}^4-\mu_2{}^4)(\mu_2-\mu_1)\\&-\frac{D_0k}{2}(\frac{\mu_2{}^4}{12}-\mu_2\frac{\mu_1{}^3}{3}+\frac{\mu_1{}^4}{4})-\frac{a_2K}{6}(t_1-\mu_2)(\mu_2{}^3-\mu_1{}^3)+\frac{b_2K}{12}(t_1{}^2-\mu_2{}^2)(\mu_2{}^3-\mu_1{}^3)-\\&D_0\{(\mu_2\mu_1-\frac{\mu_1{}^2}{2})+\frac{K}{6}(\mu_2{}^3\mu_1-\frac{\mu_1{}^4}{4})\}\}+\{a_2\frac{t_1{}^2}{2}-\frac{b_2}{3}t_1{}^3+\frac{a_2K}{12}t_1{}^4-\frac{b_2K}{15}t_1{}^5-a_2(t_1\mu_2-\frac{\mu_2{}^2}{2})-\\&\frac{a_2K}{6}(t_1{}^3\mu_2-\frac{\mu_2{}^4}{4})+\frac{b_2}{2}(t_1{}^2\mu_2-\frac{\mu_2{}^3}{3})+\frac{b_2K}{8}(t_1{}^4\mu_2-\frac{\mu_2{}^5}{5})+\frac{K}{2}a_2(t_1\frac{\mu_2{}^3}{3}-\frac{\mu_2{}^4}{4})-\frac{b_2K}{4}(t_1{}^2\frac{\mu_2{}^3}{3}-\frac{\mu_2{}^5}{5})\}\end{aligned} \tag{38}$$

The total shortage quantity during the interval [t_1, T] is:

$$B_T=-\int_{t_1}^{T}I(t).dt$$

$$B_T=\begin{aligned}&(a_2-b_2t_1)\frac{e^{-\delta t_1}}{\delta}(T-t_1)-\frac{b_2}{\delta^3}(e^{-\delta T}-e^{-\delta t_1})-\frac{b_2}{\delta^2}e^{-\delta t_1}(T-t_1)+(a_2-b_2T)\frac{e^{-\delta T}}{\delta^2}-\\&(a_2-b_2t_1)\frac{e^{-\delta t_1}}{\delta^2}-\frac{b_2}{\delta^3}(e^{-\delta T}-e^{-\delta t_1})\end{aligned} \tag{39}$$

Then the average total cost per unit time for this case is given by:-

$$C_3(t_1)=\frac{1}{T}[A_0+c_1D_T+c_2H_T+c_3B_T] \tag{40}$$

The necessary conditions for the total relevant cost per unit time of the equation (30) is to be minimized is

$$\frac{\partial C_3(t_1,T)}{dt_1}=0 \text{ and } \frac{\partial C_3(t_1,T)}{dT}=0$$

$$\left.\frac{\partial^2 C_3(t_1,T)}{\partial t_1{}^2}\right|>0, \left.\frac{\partial^2 C_3(t_1,T)}{\partial T^2}\right|>0$$

$$\left(\frac{\partial^2 C_3(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 C_3(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 C_3(t_1,T)}{\partial t_1 \partial T}\right)\Bigg| > 0 \quad (41)$$

NUMERICAL EXAMPLE

The application of an inventory model is illustrated by the following example:

$T = 12,\ \mu_1 = 5,\ \mu_2 = 10,\ a_1 = 150,\ b_1 = 5,$

$a_2 = 250,\ b_2 = 7,\ K = 0.001,\ A_0 = 250,\ c_1 = 3,\ c_2 = 0.5,\ c_3 = 5,\ \delta = 0.06$

Optimal values are:

$t_1 = 1.38972$ F = 14557.9

Table 1: Variation in total cost with the variation in K

K	t_1	**T.C**
0.0004	1.38958	14557.7
0.0006	1.38963	14557.8
0.0008	1.38967	14557.8
0.001	1.38972	14557.9
0.0012	1.38977	14557.9
0.0014	1.38981	14558
0.0016	1.38986	14558
0.0018	1.3899	14558.1
0.002	1.38995	14558.1

Table 2: Variation in total cost with the variation in a_1

a_1	t_1	**T.C.**
100	1.19764	12409.3
110	1.24414	12832.8
120	1.28598	13259.8
130	1.32383	13689.9
140	1.35826	14122.7
150	1.38972	14557.9
160	1.41858	14995.1
170	1.44517	15434.1
180	1.46974	15874.7
190	1.49252	16316.8
200	1.5137	16760.1

CONCLUSION

In the present paper, we have dealt with an EOQ inventory model for deteriorating items with shortages and trapezoidal type demand rate. It is assumed that the deterioration rate is time dependent. The nature of demand of seasonal and fashionable products is increasing-steady-decreasing and becomes asymptotic. For seasonal products like clothes, air conditions etc, demand of these items is very high at the starting of the season and become steady mid of the season and thereafter decreasing at the end of the season. The demand pattern assumed here is found to occur not only for all types of seasonal products but also for fashion apparel, computer chips of advanced computers, spare parts, etc. The procedure presented here may be applied to many practical situations. Retailers in supermarket face this type of problem to deal with highly perishable seasonal products. An approximated EOQ model is also provided by considering average demand rate over the finite time horizon. Thus, to make a better combination of increasing-steady-decreasing demand pattern for perishable seasonal products and finite length of the season this model can be used instead of assuming average demand pattern over finite time horizon. This model is more realistic for seasonal products. Cost minimization technique is used to get the expressions for total cost and other parameters. A numerical assessment of the finite planning horizon theoretical model has been done to illustrate the theory.

REFERENCES

1. Cheng, M. and Wang, G. (2008). A note on the inventory models for deteriorating items with trapezoidal type demand rate. Computer & Industrial Engineering.
2. Debata S, Acharya M, Samanta GC (2015) An inventory model for perishable items with quadratic trapezoidal type demand under partial backlogging. Int J Ind Eng Comput 6(2):185–198
3. Deng, P.S., Lin, R.H.J. and Chu, P. (2007). A note on the inventory models for deteriorating items with ramp type demand. *European Journal of Operational Research, 178, 112-120.*
4. Anand, Bansal K.K. (2013) “An Optimal Production Model or Deteriorating Item with Stocks and Price Sensitive Demand Rate” Journal of Engineering, Computers & Applied Sciences: 2(7) 32-37
5. Kumar A.,Bansal K.K.(2014) “A Deterministic Inventory Model for a Deteriorating Item is Explored in An Inflationary Environment For an Infinite Planning Horizon” International Journal of Education and Science Research Review: 1(4) 79-86
6. Garg M., Bansal K.K.“Production Model with Time Dependent Deterioration and Demand under Inflation” International Journal of Education and Science Research Review: 1(2) 1-10
7. Donaldson, W.A. (1977). Inventory replenishment policy for a linear trend in demand-An analytic solution. *Operational Research Quarterly, 28, 663-670.*
8. Giri, B.C., Jalan, A.K., Chaudhuri, K.S. (2003). Economic Order Quantity model with Weibull deterioration distribution, shortage and ramp-type demand. *International Journal of Systems Science, 34 (4), 237-243.*
9. Goyal, S.K. (1986). On improving replenishment policies for linear trend in demand. *Engineering Costs and Production Economics, 10, 73-76.*
10. Hsieh TP, Dye CY (2013) A production inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. J Comput Appl Math 239(1):25–36
11. Khanra, S., Chaudhuri, K.S. (2003). A note on an order-level inventory model for a deteriorating item with time-dependent quadratic demand. *Computers and Operations Research, 30, 1901-1916.*
12. Kouki C, Sahin E, Jemai Z, Dallery Y (2013) Assessing the impact of perishability and the use of time temperature technologies on inventory management. Int J Prod Econ 143(1):72–85
13. Manna, S.K., Chaudhuri, K.S. (2006). An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages. *European Journal of Operational Research, 171, 557-566.*
14. Silver, E. A., Meal. H.C. (1969). A simple modification of the EOQ for the case of a varying demand rate. *Production and Inventory Management, 10(4), 52-65.*
15. Singh SR, Gupta V (2014) Supply chain production model with preservation technology under fuzzy environment. Int J Ind Eng Comput 5(3):459–474
16. Wu, K.S. (2001). An EOQ model for items with Weibull distribution deterioration, ramp-type demand rate and partial backlogging. Production Planning & Control, 12 (8), 787-793.
17. Wu, K.S., Ouyang, L. Y. (2000). A replenishment policy for deteriorating items with ramp type demand rate for deteriorating items. *The Proceedings of the National Science council, Part A: Physical Science Engineering, 24(4), 279-286.*
18. Zhao L (2014) An inventory model under trapezoidal type demand, Weibull distributed deterioration, and partial backlogging. J Appl Math. Article ID 747419.